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Quantum-vacuum geometric phases in the noncoplanarly curved fiber system

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Abstract. The connection between the *quantum-vacuum geometric phases* (which originates from the vacuum zero-point electromagnetic fluctuation) and the *non-normal order* for operator product is considered in the present paper. In order to investigate this physically interesting geometric phases at quantum-vacuum level, we suggest an experimentally feasible scheme to test it by means of a noncoplanarly curved fiber made of gyrotropic media. A remarkable feature of the present experimental realization is that one can easily extract the nonvanishing and nontrivial quantum-vacuum geometric phases of left- and/or right-handed circularly polarized light from the vanishing and trivial total quantum-vacuum geometric phases. Since the normal-order procedure may remove globally the vacuum energy of time-dependent quantum systems, the potential physical vacuum effects (e.g., quantum-vacuum geometric phases) may also be removed by this procedure. Thus the detection of the geometric phases at quantum-vacuum level may answer whether the normal-order technique is valid or not in the time-dependent quantum field theory.

PACS. 03.65.Vf Phases: geometric; dynamic or topological – 03.70.+k Theory of quantized fields – 42.70.-a Optical materials – 42.50.Xa Optical tests of quantum theory

1 Introduction

Since Berry discovered that a topological (geometric) phase exists in quantum mechanical wavefunction of time-dependent systems, geometric phase problems have captured considerable attention of researchers in various fields, including quantum mechanics [1], differential geometry [2], gravity theory [3], atomic and molecular physics [4,5], nuclear physics [6], quantum optics [7], condensed matter physics [8], molecular systems and chemical reaction [4] as well. Differing from dynamical phase that depends on dynamical quantities (such as energy, frequency, velocity and coupling coefficients as well) of systems, geometric phase is independent of these dynamical quantities. Instead, it is only related to the geometric nature of the pathway along which the quantum systems evolve. It has been well-known that geometric phase arises in systems with the time-dependent Hamiltonian, or in systems the Hamiltonian of which possesses some evolution parameters. This, therefore, implies that geometric phase presents the topological and global properties of quantum systems in time-development process, and that it possesses the physical significance and can thus be employed to various fields of physics [4–8]. More recently, many authors concentrated their particular attention on the potential applications of geometric phases to the geometric quantum computation, quantum decoherence and related topics [9–12]. One of the most important physical realizations of Berry's phase (i.e., cyclic adiabatic geometric phase) is the model describing the propagation of photons inside a helically curved optical fiber, which was proposed by Chiao and Wu [13], and later performed experimentally by Tomita and Chiao [14]. Afterwards, a large number of investigators treated this photon geometric phases by making use of the classical Maxwell's electrodynamics, differential geometry method (parallel transport) and quantum adiabatic theory both theoretically and experimentally [15–18]. Based on the above investigations, we studied the nonadiabatic noncyclic geometric phases of photons propagating inside a noncoplanarly curved optical fiber [19,20] by means of the Lewis-Riesenfeld invariant theory [21] and the invariant-related unitary transformation formulation [22]. By using the obtained results [19,20], we considered the photon helicity inversion in the curved fiber and its potential

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applications to information science [23] and proposed a second-quantized spin model to describe the coiled light in a curved fiber, where the vacuum zero-point fluctuation is involved. As was stated by Fuentes-Guridi et al., in a strict sense, the Berry phase has been studied only in a semiclassical context until now [24]. Thus the effects of the vacuum field on the geometric evolution are still unknown [24]. In their paper [24], Fuentes-Guridi et al. considered the time evolution of a spin-1/2 particle interacting with a second-quantized external magnetic field and proposed a vacuum-induced spin-1/2 Berry's phase, which they regarded as the effect of vacuum photon fluctuation. It should be noted that the contribution of such a quantum electromagnetic fluctuation to geometric phases was also suggested independently by us in the papers [23, 25], where we briefly considered the effect of quantum vacuum fluctuation on the photon wavefunction propagating inside a noncoplanarly curved fiber [23] and showed that the photon wavefunction will acquire a so-called quantum-vacuum geometric phase. The essential difference between Fuentes-Guridi et al.'s vacuum-induced Berry's phase [24] and our quantum-vacuum geometric phase [23,25] lies in that the former phase belongs to the spin-1/2 charged particle experiencing an interaction with a quantized external magnetic field, while the latter phase belongs to the photon field itself which undergoes an "inertial force" in a curved fiber [23]. This means that in Fuentes-Guridi et al.'s work the spin-1/2 particle is actually the protagonist [24], whereas in our work the protagonist under consideration is just the second-quantized electromagnetic field itself [23,25]. In other words, the quantum-vacuum geometric phase of photons considered by us is a purely optical effect at the vacuum level. In this paper, we will study further this nontrivial vacuum effect, and consider its novel properties (particularly its connection with the normal-order procedure in quantum field theory), and then suggest a scheme of experimental realization of this geometric phases at quantum-vacuum level by using the gyrotropic-medium fiber.

This paper is organized as follows: in Section 2, we discuss the necessity of non-normal product in the time-dependent quantum field theory; in Section 3, we give the expressions for the quantum-vacuum geometric phases of left- and right-handed circularly polarized light, and then show why the total vacuum geometric phases of polarized light is generally absent in experiments [14–18]. In Section 4, we propose a scheme to detect the nonvanishing vacuum geometric phases of one (left- or right-handed) of polarized light. In Section 5, we consider the potential physical significance of quantum-vacuum geometric phases in quantum mechanics and time-dependent quantum field theory.

2 Non-normal order and observable vacuum effects

Note that here the quantum-vacuum geometric phases of photons results from the zero-point energy of vacuum quantum fluctuation. This, therefore, means that this geometric phases is quantal in character and, moreover, has no classical counterpart, namely, it cannot survive the correspondence-principle limit into the classical level. It is well-known that in the conventional quantum field theory, both infinite vacuum fluctuation energy and divergent vacuum electric charge density are removed by the so-called *normal-order procedure* and the new vacuum backgrounds of quantum fields in which the vacuum expectation values of both charge density and Hamiltonian vanish, are therefore re-defined. Since in the *time-independent* field theory, the infinite constant is harmless and easily removed, the normal-order procedure applied to these *time-independent* cases is reliable and valid indeed. However, in the *time-dependent* quantum field theory (such as quantum field theory in curved space-time, e.g., time-dependent gravitational backgrounds and expanding universe), the time-dependent vacuum zero-point fields itself may also participate in the time evolution process and therefore cannot be regarded merely as an inactive onlooker. In order to clarify this point, we will discuss the above problem in more details.

In the time-independent quantum field theory, the vacuum fluctuation energy has been removed by the normal-order procedure. Such a technique is harmless since we remove the same amount of background energy at different time (i.e., the vacuum background energy is removed *globally*). However, in the time-dependent field theory, if we use the normal-order procedure, we will remove the different amount of background energy at different time in the evolution process (i.e., the vacuum background energy is removed based on the different cardinal numbers at different time). Thus the vacuum background is so redefined by removing different amount of zero-point energies in the time-development process, and then some observable vacuum effects may also be cancelled theoretically by this normal-order procedure. Since in the time-dependent quantum systems, one of the most attractive and important effects is the geometric phases, we should consider the connection between the non-normal order and the geometric phases at quantum-vacuum level.

In the above, we have shown that the quantum-vacuum geometric phases may be removed by the normal-order procedure. However, geometric phases is a physically interesting effect. This, therefore, means that the normal-order procedure will inevitably remove the physical effects. Such a situation is not quite satisfactory. However, the detection of the existence of vacuum geometric phases may answer whether the normal-order technique is valid or not in the time-dependent quantum field theory. If we cannot test the vacuum geometric phases experimentally, then we can conclude that the normal-order technique is still valid in the time-dependent quantum field theory. But if experimental evidences show that there truly exists the quantum-vacuum geometric phases, then we may argue that the normal-order technique might be no longer valid in the time-dependent quantum field theory. In this sense, the quantum-vacuum geometric phases may be regarded as just a touchstone for the normal-order procedure.

For this aim, in the *time-dependent* cases, we should first take into account the non-normal order for operator product in quantum field theory and subsequently predict some physically interesting vacuum effects associated with time-dependent quantum fluctuation. In what follows by analyzing the time evolution of photon wavefunction we will consider such vacuum effects in a time-dependent quantum system, i.e., the Tomita-Chiao-Wu fiber system [13,14,19] where the rotation of photon polarization planes, which gives rise to photon geometric phases, in a noncoplanar fiber takes place.

3 Quantum-vacuum geometric phases in noncoplanarly curved fibers

In this section, we will demonstrate theoretically how the quantum-vacuum geometric phase of photons arises in the Tomita-Chiao-Wu fiber system [13,14] when the normal order procedure is not applied to the derivation of the geometric phases and wavefunctions of photons moving in a noncoplanar optical fiber. According to the Liouville-von Neumann equation [19], where the Lewis-Riesenfeld invariant [21] is the photon helicity $h = \mathbf{k}\cdot\mathbf{S}/k$, the effective Hamiltonian that describes the propagation of coiled light in a curved fiber is of the form [19,20] $H_{\rm eff}(t) = [\mathbf{k}(t)\times\dot{\mathbf{k}}(t)]\cdot\mathbf{S}/k^2$ with $\mathbf{k}$, k and $\mathbf{S}$ representing the wave vector, the magnitude of $\mathbf{k}$ and the spin operator of photons. The dot denotes the derivative of $\mathbf{k}$ with respect to time. Thus the time-dependent Schrödinger equation governing the time evolution of photon wavefunction inside the noncoplanarly curved fiber is written in the form [19,20] (in the unit $\hbar = c = 1$)

$$i\frac{\partial\left|\sigma,\mathbf{k}(t)\right\rangle}{\partial t} = \frac{\mathbf{k}(t)\times\dot{\mathbf{k}}(t)}{k^2}\cdot\mathbf{S}\left|\sigma,\mathbf{k}(t)\right\rangle, \qquad (3.1)$$

where $\sigma = \pm 1$ is the eigenvalues of photon helicity h corresponding to the right- and left-handed circularly polarized photons. The photon wave vector $\mathbf{k}$ inside the fiber can be written in the spherical polar coordinate system, i.e., $\mathbf{k} = k(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, which is always along the tangent to the curved fiber at each point at arbitrary time. By making use of the Lewis-Riesenfeld invariant theory and the invariant-related unitary transformation formulation [21,22], we obtain the exact particular solutions $|\sigma,\mathbf{k}(t)\rangle = \exp[(1/i)\phi_\sigma^{(\rm g)}(t)]V(t)|\sigma,k\rangle$ to equation (3.1), where $|\sigma,k\rangle \equiv |\sigma,\mathbf{k}(t=0)\rangle$ is the initial polarized photon state, $V(t) = \exp[\beta(t)\mathbf{S}_+ - \beta^*(t)\mathbf{S}_-]$ with $\beta(t) = -[\theta(t)/2]\exp[-i\varphi(t)]$, $\beta^*(t) = -[\theta(t)/2]\exp[i\varphi(t)]$ [19] and $\mathbf{S}_\pm = \mathbf{S}_1 \pm i\mathbf{S}_2$. In this paper, we consider only the case where the polar angle θ is constant (i.e., the nutational frequency of photon wave vector $\mathbf{k}$ vanishes) and the azimuthal angle φ changes just with time (i.e., the precessional frequency $\dot\varphi$ is constant, ω). Thus the noncyclic geometric phase is given as follows

$$\phi_\sigma^{(\rm g)}(t) = \Omega(t)\left\langle\sigma,k\right|S_3\left|\sigma,k\right\rangle \qquad (3.2)$$

with $\Omega(t) = \arctan(\tan\omega t\cos\theta) - \omega t\cos\theta$ [26,27]. It is apparent that in the cyclic case for the Chiao-Wu's coiled light in a helically curved fiber, where $\dot\theta = 0$ and the rotating frequency of photon moving on the fiber helicoid is ω, the geometric phase in a cycle ($T = 2\pi/\omega$) over the photon momentum space takes the form $\phi_\sigma^{(\rm g)}(T) = 2\pi(1-\cos\theta)\left\langle\sigma,k\right|S_3\left|\sigma,k\right\rangle$, where the expression $2\pi(1-\cos\theta)$ denotes the solid angle subtended by a curve traced by the wave vector at the center of photon momentum space. This fact demonstrates the topological and global properties of geometric phases. This shows that the above calculation is self-consistent.

Now we consider the expectation value, $\left\langle\sigma,k\right|S_3\left|\sigma,k\right\rangle$, of the third component of photon spin operator in equation (3.2). Substitution of the Fourier expansion series of three-dimensional magnetic vector potentials $\mathbf{A}(\mathbf{x},\mathbf{t})$ into the expression $S_{ij} = -\int(\dot{A}_iA_j - \dot{A}_jA_i){\rm d}^3\mathbf{x}$ for the spin operator of photon fields yields

$$S_3 = \frac{i}{2}[a(k,1)a^\dagger(k,2) - a^\dagger(k,1)a(k,2) - a(k,2)a^\dagger(k,1) + a^\dagger(k,2)a(k,1)] \qquad (3.3)$$

with $a^\dagger(k,\lambda)$ and $a(k,\lambda)$ ($\lambda = 1,2$) being the creation and annihilation operators of polarized photons corresponding to the two mutually perpendicular real unit polarization vectors. Note that here S_3 is of the non-normal-order form.

In what follows we define the creation and annihilation operators, $a_R^\dagger(k)$, $a_L^\dagger(k)$, $a_R(k)$, $a_L(k)$, of right- and left-handed circularly polarized light [28], which are expressed in terms of $a^\dagger(k,\lambda)$ and $a(k,\lambda)$, i.e., $a_R^\dagger(k) = 1/\sqrt{2}[a^\dagger(k,1)+ia^\dagger(k,2)]$, $a_R(k) = 1/\sqrt{2}[a(k,1)-ia(k,2)]$, $a_L^\dagger(k) = 1/\sqrt{2}[a^\dagger(k,1)-ia^\dagger(k,2)]$ and $a_L(k) = 1/\sqrt{2}[a(k,1)+ia(k,2)]$. Thus equation (3.3) can be rewritten in terms of the creation and annihilation operators of right- and left-handed polarized photons, namely,

$$S_3 = \frac{1}{2}[a_R(k)a_R^\dagger(k) + a_R^\dagger(k)a_R(k)] - \frac{1}{2}[a_L(k)a_L^\dagger(k) + a_L^\dagger(k)a_L(k)], \qquad (3.4)$$

which can also be rewritten as $S_3 = [a_R^\dagger(k)a_R(k) + 1/2] - [a_L^\dagger(k)a_L(k) + 1/2]$.

The monomode multiphoton states of left- and right-handed (LRH) circularly polarized light (at $t = 0$) can be defined as $|\sigma=-1,k,n_L\rangle = (n_L!)^{-1/2}[a_L^\dagger(k)]^{n_L}|0_L\rangle$ and $|\sigma=+1,k,n_R\rangle = (n_R!)^{-1/2}[a_R^\dagger(k)]^{n_R}|0_R\rangle$ with n_L and n_R being the LRH polarized photon occupation numbers, respectively. In the following we will calculate the total geometric phases of monomode multiphoton states $|\sigma=+1,k,n_R;\sigma=-1,k,n_L\rangle$, which is the direct product of LRH polarized multiphoton states, i.e., $|\sigma=+1,k,n_R\rangle\otimes|\sigma=-1,k,n_L\rangle$. Insertion of the expression for the monomode multiphoton states into equation (3.2) yields the geometric phases of multiphoton polarized states, $\phi^{(\rm g)}(t) = (n_R - n_L)\Omega(t)$, which is independent of k but dependent on the geometric nature of the pathway (expressed in terms of θ and φ) along which the light wave propagates. Although the phases $\phi^{(\mathrm{g})}(t)$ associated with the photonic occupation numbers n_R and n_L are quantal geometric phases of photons [20], they do not belong to the geometric phases at quantum-vacuum level which arises, however, from the zero-point electromagnetic energy of vacuum quantum fluctuation.

It follows from expression (3.4) for S_3 that the separate geometric phases (different from the above total geometric phases $\phi^{(\mathrm{g})}(t)$) of left- and right-handed circularly polarized photon states, i.e., $|\sigma = -1, k, n_L\rangle$ and $|\sigma = +1, k, n_R\rangle$, are respectively as follows

$$\phi_L^{(\mathrm{g})}(t) = -\left(n_L + \frac{1}{2}\right)\Omega(t),$$

$$\phi_R^{(\mathrm{g})}(t) = +\left(n_R + \frac{1}{2}\right)\Omega(t). \quad (3.5)$$

Note again that in Tomita-Chiao experiment [14], $\Omega(t)$ in (3.5) is the solid angle, $\Omega(T)$=$2\pi(1-\cos\theta)$, traced by the photon wave vector at the center.

It follows from equation (3.5) that the total geometric phases of circularly polarized light is

$$\phi_{\mathrm{total}}^{(\mathrm{g})}(t) = \left[\left(n_R + \frac{1}{2}\right) - \left(n_L + \frac{1}{2}\right)\right]\Omega(t), \quad (3.6)$$

which can be rewritten as

$$\phi_{\mathrm{total}}^{(\mathrm{g})}(t) = (n_R - n_L)\,\Omega(t). \quad (3.7)$$

It should be noted that even though the expression (3.6) is mathematically equivalent to (3.7), there are essential differences in physical meanings between them. Namely, the expression (3.6) contains the geometric phases at quantum-vacuum level, while the expression (3.7) does not involve such a vacuum effect. In the papers [19,20], the authors studied the photon geometric phases (3.7) in a fiber system. In the present paper, we concentrate our attention more on the geometric phases (3.6) and its experimental realization.

According to equation (3.5), it is readily verified that the time-dependent zero-point energy possesses physical meanings and therefore contributes to geometric phases of photon fields. Thus the noncyclic nonadiabatic geometric phases of left- and right-handed polarized states at quantum-vacuum level are of the form

$$\phi_{\sigma=\pm 1}^{(\mathrm{vac})}(t) = \pm\frac{1}{2}\Omega(t). \quad (3.8)$$

However, it should be pointed out that, unfortunately, even at the quantum level, this quantum-vacuum geometric phases $\phi_{\sigma=\pm 1}^{(\mathrm{vac})}(t)$ that is observable in principle is absent in the previous fiber experiments [14–17], since it follows from (3.5) and (3.8) that the signs of quantal geometric phases of left- and right-handed circularly polarized photons are just opposite to one another, and so that their quantum-vacuum geometric phases (3.8) are counteracted by each other. Hence the observed geometric phases are only those associated with the creation operators $a_L^\dagger$ and $a_R^\dagger$ of LRH polarized photons, the cyclic adiabatic case of which has been measured in the optical fiber experiments performed by Tomita and Chiao et al. [14–17]. Although the total of LRH quantum-vacuum geometric phases (3.8) is trivial, the separate vacuum geometric phase of circularly polarized field is nontrivial, which deserves experimental investigation. The troublesome problem left to us now is that how can we detect the above quantum-vacuum geometric phases of left- and/or right-handed polarized fields that has been cancelled by each other?

4 A realization of quantum-vacuum geometric phases

More recently, we suggest a new scheme to test the existence of this vacuum effect, the idea of which is to extract the nonvanishing cyclic quantum-vacuum geometric phases $\phi_{\sigma=+1}^{(\mathrm{vac})}(T)$ or $\phi_{\sigma=-1}^{(\mathrm{vac})}(T)$ by changing the mode distribution structures of vacuum photon field (or inhibiting the vacuum photon fluctuation of certain propagation constant). This is not strange to us. For example, it is well-known that in Casimir's effect the vacuum-fluctuation electromagnetic field in a finitely large space (i.e., the space between two parallel metallic planes) will alter its mode structures, namely, the zero-point field with wave vector k less than $\sim(\pi/a)$ does not exist in this surrounding space with a finite scale length a. Another illustrative example is the inhibition or enhancement of spontaneous emission in photonic crystals [29] and resonant cavity [30], where the vacuum modes at certain transition frequencies is drastically modified as compared to that in free space.

For this aim, we take into account the peculiar wave propagation inside a kind of anisotropic materials (gyrotropic media), the electric permittivity and magnetic permeability of which are tensors taking the following form [31]

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \quad (4.1)$$

Assuming that the direction of the electromagnetic wave vector $\mathbf{k}$ is parallel to the third component of the Cartesian coordinate system, with the help of Maxwell's equations, one can arrive at [31,32]

$$n_\pm^2 = (\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2), \quad (4.2)$$

where n_+ and n_- are the optical refractive indices of such gyrotropic media corresponding to the right- and left-handed circularly polarized light, respectively [31,32]. Since in such gyrotropic media, if one is positive and the other negative for the optical refractive indices squared n^2 corresponding to the two directions of polarization of the electromagnetic wave, then only one wave can propagate in this gyrotropic media.

We think that there may be two ways of changing the density in the distribution of modes of vacuum fluctuation fields: one is the change of boundary conditions (e.g., Casimir effect), the other is the choice of appropriate optical constants (electric permittivity, magnetic permeability and refractive index) of media. It is truly understandable that on the classical level the electromagnetic wave can be inhibited by the dissipative media. We believe that, likewise, the electromagnetic media may also have influence on the mode density of vacuum quantum fluctuation. In order to clarify this point, here we discuss two illustrative examples:

(i) photonic crystals are one of the artificial materials patterned with a periodicity in dielectric constant, which can create a range of forbidden frequencies called the photonic band gap (PBG). It is well-known that in the photonic crystals the rates of spontaneous emission of atoms will be influenced greatly by the PBG structures. Recently, Klimov investigated theoretically the modified spontaneous emission rates in the left-handed media, which possess the permittivity and permeability with negative real parts (and hence the negative real part of the optical refractive index) [33]. Since the spontaneous emission of atoms results from the interaction between the excited atoms and the vacuum zero-point electromagnetic field, these investigations show that the unusual properties of optical constants (including the spatial variation, complex function, and the corresponding band gap structures) may lead to some changes of the density in the distribution of modes of vacuum fluctuation fields;

(ii) more recently, Dung et al. studied in detail the problems of the electromagnetic-field quantization and the spontaneous emission in left-handed media. Their work shows that the spontaneous decay rate of an excited atom will be influenced by the dispersing and absorbing magnetodielectric media having complex optical "constants" with the negative real parts [34].

In view of the above work of spontaneous emission and field quantization in artificial electromagnetic media [33,34], it is believed that the unusual changes of optical constants of media will inevitably have effects on the density in the distribution of modes of both the classical fields and the vacuum fluctuation fields inside these media. So, the vacuum contribution of one of the two polarizations may possibly be reduced by the gyrotropic media with appropriate electromagnetic parameters $\epsilon_{1,2}$ and $\mu_{1,2}$ in the permittivity and permeability tensors (4.1). Thus, the quantum-vacuum geometric phases of LRH polarized photons cannot be eliminated by each other and it is therefore possible for physicists to easily test the remainder of them experimentally. If, for example, by taking some certain values of ϵ_1, ϵ_2, μ_1 and μ_2, then $n_-^2 < 0$ while $n_+^2 > 0$ and consequently the left-handed polarized light cannot be propagated in this medium, and in the meanwhile the quantum vacuum fluctuation corresponding to the left-handed polarized light will also be inhibited (e.g., the wave amplitude exponentially decreases because of the imaginary part of the refractive index n_-) in this anisotropic absorptive medium. Thus the vacuum-fluctuation electromagnetic field alters its mode structures in such an absorptive medium. For this reason, the only retained geometric phase is that of right-handed polarized light, which we can detect experimentally.

As an illustrative example, we now discuss the light propagation inside an optical fiber made of gyrotropic media. We only consider the condition under which $|\epsilon_2| > |\epsilon_1| = -\epsilon_1$ and $\mu_1 \pm \mu_2 > 0$. If, for instance, ϵ_2 is positive, then the right-handed polarized light can be propagated while the left-handed polarized light cannot be propagated in the fiber (because of the negative n_-^2 and the consequent imaginary propagation constant k_-, which is expressed by $n_-\omega/c$); conversely, if ϵ_2 is negative, then the left-handed polarized light can be propagated while the right-handed polarized light is inhibited from being propagated (due to the imaginary propagation constant k_+, which equals $n_+\omega/c$). Thus in the former case the phase $\phi_R^{(\mathrm{vac})}(T)$ of right-handed polarized light, and in the latter case the phase $\phi_L^{(\mathrm{vac})}(T)$ of left-handed polarized light instead, may respectively be detected in this gyrotropic-medium fiber experiment.

Since the vacuum photon fluctuation with k less than π/a will be inhibited in the space between two parallel conducting plates whose separation is a, we can suggest another scheme to detect $\phi_{\sigma=\pm1}^{(\mathrm{vac})}(T)$: specifically, if ϵ_1 and ϵ_2 (or μ_1 and μ_2) of gyrotropic medium are chosen to be $\epsilon_1 = \epsilon_2$ (or $\mu_1 = \mu_2$), then the vacuum fluctuation corresponding to the left-handed polarized light is inhibited since its propagation constant k_- tends to zero (and hence the wavelength is much larger than the space scale a). Thus the only retained vacuum geometric phase is that of right-handed polarized light.

5 Physical significance of quantum-vacuum geometric phases

Historically, field theory encountered problems such as divergent zero-point energies of quantized electromagnetic fields and infinite electric charge density arising from the presence of electrons of negative energies [28]. These problems can be solved by applying the normal-order technique to the field operators. Thus both the background charges and vacuum fluctuation energies are removed (i.e., the vacuum expectation values of both charge density and Hamiltonian vanish). In such systems of field theory, the Hamiltonian of which is *time-independent*, the same amount of zero-point energy is eliminated in the evolution process, which is equivalent to re-defining the background energies with the same base number at different time. Hence, in these cases the normal-order procedure is practical and valid. However, whether the normal product is valid or not should still be taken into consideration for systems with *time-dependent* Hamiltonians. Moreover, we think that in the time-dependent field theory the validity of the above formulation may deserve much incredulity. In view of the above remarks, it is emphasized that investigations of vacuum states of time-dependent systems may become particularly important. So, we think that the test of the abovementioned quantum-vacuum geometric phases in the fiber system may be of imperative necessity.

The physical significance of the subject presented in this paper may be as follows:

(i) the quantum-vacuum geometric phases found here possesses interesting properties, for it has an important connection with the topological nature of time evolution of quantum vacuum fluctuation. To the best of our knowledge, in the literature, less attention was paid to such geometric phases at purely quantum-vacuum level. According to Fuentes-Guridi's statement, such vacuum geometric phases may open up a new areas to the study of the consequences of field quantization in the geometric evolution of states [24], so, it is emphasized that this vacuum effect deserves further consideration both theoretically and experimentally;

(ii) in order to extract the nontrivial quantum-vacuum geometric phases of polarized light from the trivial total quantum-vacuum geometric phases (which has been cancelled by each other and therefore vanishing), a new scheme, which is somewhat ingenious, by using the gyrotropic-medium optical fiber is proposed;

(iii) we think that the detection of quantum-vacuum geometric phases may be essential for the investigation of the *time-dependent* quantum field theory. As was discussed above, the quantum-vacuum geometric phases is related close to the non-normal order for operator product in second quantization. If the existence of quantum-vacuum geometric phases is truly demonstrated in experiments, then we should consider the validity problem of normal product technique in the time-dependent field theory. This, therefore, means that the experimental study of quantum-vacuum geometric phases might become a fundamental and important subject in quantization formulation and time-dependent field theory.

Recently, many authors applied geometric phases to some areas such as quantum decoherence and geometric (topological) quantum computation [9–12]. It may be believed that the quantum-vacuum geometric phases in the fiber will probably also have some possible interesting applications to these subjects. We hope all the effects and phenomena presented in this paper would be investigated experimentally in the near future.

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